Yerbilimleri, 2016, 37 (3), 271-284
Hacettepe Üniversitesi Yerbilimleri Uygulama ve Araştırma Merkezi Bülteni
Bulletin of the Earth Sciences Application and Research Centre of Hacettepe University

Analytic Notch Filter Design Using the Hyperbolic Secant Function

Hiperbolik Sekant Fonksiyonlar ile Analitik Çentik Süzgeç Tasarımı

ERTAN PEKŞEN [1*], FATİH SEVİM [2]
[1]Kocaeli University, Faculty of Engineering, Department of Geophysical Engineering, İzmit, Kocaeli
[2]TÜBİTAK MAM Gebze, Kocaeli

Geliş *(received)* : 15 Şubat (*February*) 2016
Kabul *(accepted)* : 15 Ekim (*October*) 2016

ABSTRACT
A notch filter can be analytically designed by using the hyperbolic secant function. This paper investigates notch filters in both time and frequency domains. Generally, the purpose of a notch filter design is to filter some unwanted signal (usually interference at 50 or 60 Hz) from observed data. In this study, the impulse response of a notch filter in the time domain was obtained by using its frequency domain expression. A cascaded-notch filter was analytically designed as well. Numerical examples were considered for a single notch filter and a cascaded-notch filter in the time and frequency domain. The frequency response and the impulse response of proposed notch filter were derived. Finally, the new designed filter was successfully applied to a field data set.

Keywords: Cascaded Notch Filter, Hyperbolic Secant Function, Notch Filter, Seismograms.

ÖZ
Hiperbolik sekant fonksiyonlar ile çentik süzgeç tasarlanabilir. Bu çalışmada zaman ve frekans ortamlarında çentik süzgeçler incelenmektedir. Genel olarak, çentik süzgeç tasarımının amacı ölçülen verilerden istenmeyen (genellikle 50 veya 60 Hz lik girişimler) kısımların süzülmesidir. Bu çalışmada zaman ortamındaki çentik süzgeç fonksiyonu, çentik süzgeç fonksiyonunun frekans ortamındaki ifadesinden türetilmiştir. Basamaklı çentik süzgeci de analitik olarak tasarlanmıştır. Sayısal örneklerde çentik ve basamaklı çentik süzgeçler zaman ve frekans ortamında dikkate alınmıştır. Önerilen çentik süzgecin birim dürtü ve frekans tepki bağıntıları türetilmiştir. Son olarak, yeni tasarlanan filtre başarılı bir şekilde arazi verisine uygulanmıştır.

Anahtar Kelimeler: *Basamaklı Çentik Süzgeci , Hiperbolik Sekant Fonksiyon, Çentik Süzgeci, Sismogram.*

*E. Pekşen
e-posta: ertanpeksen@kocaeli.edu.tr

INTRODUCTION

The design of a digital notch filter is important in geophysics, electronics, biomedical sciences and other sciences. In geophysics, measured field data can be often contaminated by the frequency of power-line, therefore, the frequencies of power lines and from other sources need to be filtered out from the measured field data to make the measured data contaminated free – signals. An ideal notch filter is used in removing the power-line frequency from seismograms, or some other types of signals.

There have been numerous publications about filter design in literature (e.g. Deshpande et al., 2008; Piskorowski, 2013; Stancic and Nikolic, 2013; Piskorowski, 2012). There are many filter designing options, for instance, a notch filter can be designed with z-Transform method (pole-zero placement method) (Cadzow, 1973), and hyperbolic tangent function can be used for designing various digital filters (Johensen and Sorensen, 1979; Başokur, 1998; Başokur, 2011). A notch filter is not the only method use in filtering out a single harmonic from a data set, subtraction noise from signal can also be used for filtering purpose (Butler and Russell, 1993).

Since the power-line frequency is known e.g. 50 Hz or 60 Hz, these frequencies can be subtracted from geophysical records. However, the disadvantage of this method is that, the power-line frequency is not always at 50 Hz or 60 Hz. There are always some deviations of the mean value of the power-line frequency (Figure 1). To get rid of these sorts of stochastic noise from the corresponding data, adaptive filtering methods may be applied (Butler and Russell, 1993; Hattingh, 1988; Haykin, 1991).

The main aim of this work is to develop an analytic digital notch filter. This paper is organized in three sections as follows; the first section contains the derivation of an analytic expression for the notch filter and numerical examples of the notch filter. In the second part, demonstration of how this method can be applied to a cascaded-notch filter is shown. And in the final section, the presented notch filter was applied to a field data set.

DIGITAL NOTCH FILTER

Notch Filter Design

The behavior of the hyperbolic secant function is shown in Figure 2. To obtain an analytic expression for a notch filter, the Fourier pairs of the hyperbolic secant function can be used. The Fourier pair of the hyperbolic secant function is given by Gradshteyn and Ryzhik (1994) as shown below;

$$n(t) = \operatorname{sech}(c\,t) \leftrightarrow \frac{\pi}{c}\operatorname{sech}\left(\frac{\pi^2 f}{c}\right) = N(f) \qquad (1)$$

where *t* is time (s) and f is a frequency (Hz). *c* is a constant, which mainly controls the slope of the function. Figure 2 displays the behavior of the hyperbolic secant function in the frequency domain for various c values. It could be observed that, increasing in the value of c, decreases the slope of the hyperbolic secant function. Thus, it is possible to choose an optimum *c* for filtering out an unwanted signal from a given data and to control the band width. The lower panel of Figure 2 illustrates normalized amplitudes.

Using the Fourier pairs of Equation 1 and combining the hyperbolic secant functions gives

$$H(f) = \left(\frac{\pi}{c}\right)\operatorname{sech}\left(\frac{\pi^2(f+f_c)}{c}\right) + \left(\frac{\pi}{c}\right)\operatorname{sech}\left(\frac{\pi^2(f-f_c)}{c}\right) \qquad (2)$$

where f_c is the cut-off frequency. If Equation 2 is subtracted from a constant, it yields

$$H(f) = \frac{\pi}{c} - \left\{\left(\frac{\pi}{c}\right)\operatorname{sech}\left(\frac{\pi^2(f+f_c)}{c}\right) + \left(\frac{\pi}{c}\right)\operatorname{sech}\left(\frac{\pi^2(f-f_c)}{c}\right)\right\} \qquad (3)$$

Equation 3 is the frequency response of the notch filter. Figure 3 demonstrates how the hyperbolic secant functions behave in the frequency domain. Figure 3 (a) displays a constant value. Figure 3 (b) illustrates the behaviors of the filter characteristic. Finally, the last panel of the Figure 3 (c) shows the behavior of a desired notch filter. As seen in Figure 3, the frequency response of the hyperbolic secant function notches the signal at the cut-off frequency, which is at 15 Hz. In this example, c is chosen as 5.

Equation 3 can be rewritten as

$$H(f) = A - \left\{N(f+f_c) + N(f-f_c)\right\} \qquad (4)$$

where $A = \pi/c$. The shift theorem for the Fourier transform (Bracewell, 1965) may be used for deriving an expression for the notch filter. Thus, Equation 4 can then be written in the form

$$h(t) = A\delta(t) - \left\{e^{-i2\pi f_c t}n(t) + e^{i2\pi f_c t}n(t)\right\} \qquad (5)$$

where $\delta(t)$ is the Dirac delta function. Consider $e^{\pm i2\pi f_c t} = \cos(2\pi f_c t) \pm i\sin(2\pi f_c t)$ then substitute the Euler identity into Equation 5, results in

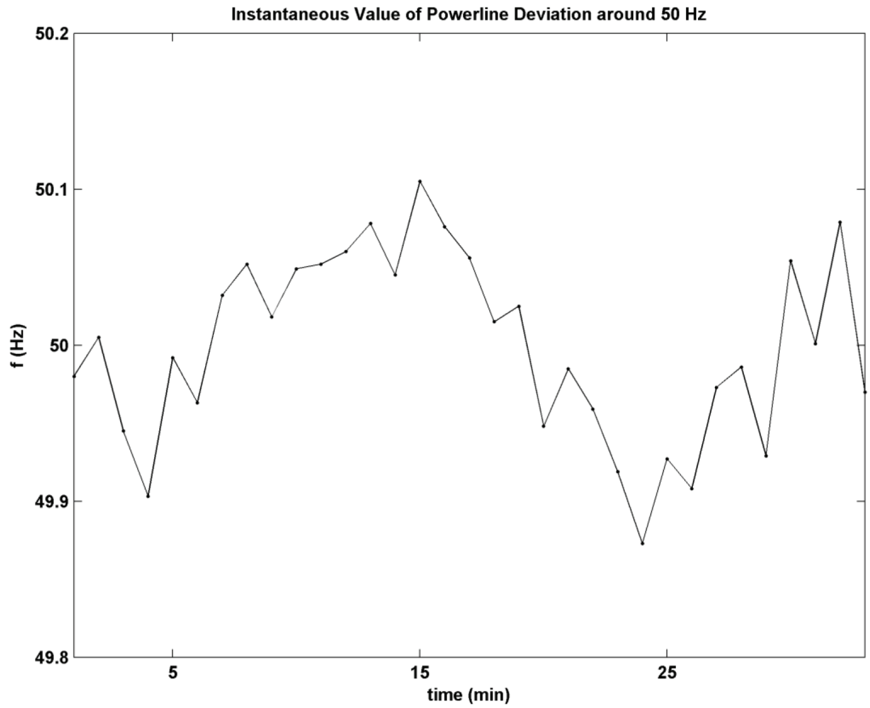

Figure 1. The curve illustrates instantaneous value of power-line frequency variation measured outlet of a building.
Şekil 1. Eğri, bir bina içindeki elektrik prizinde ölçülen anlık güç hattı frekans değişimini göstermektedir.

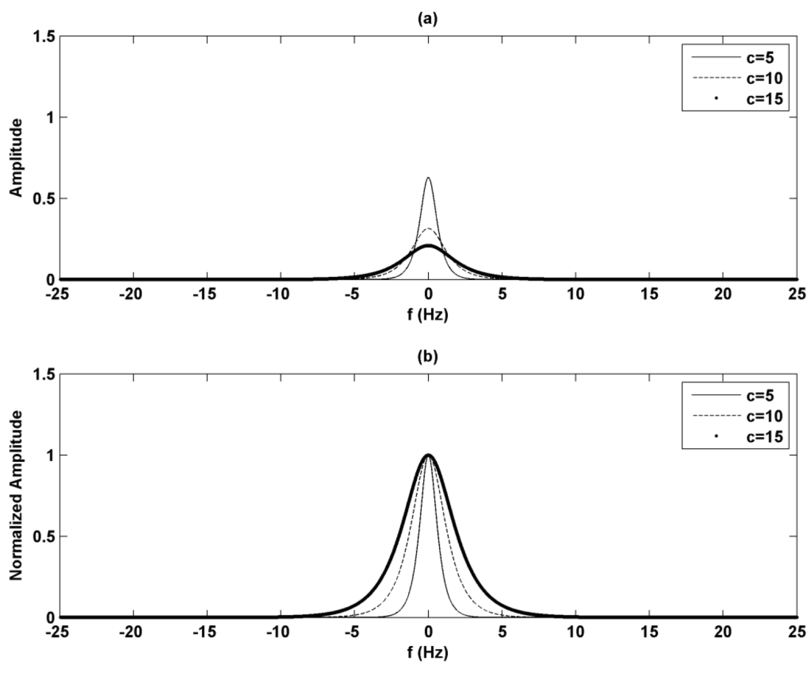

Figure 2. (a) The behavior of secant hyperbolic function for various c values. (b) Normalized secant hyperbolic function.
Şekil 2. (a) Farklı c değerlerine göre sekant hiperbolik fonksiyonun değişimi. (b) Normalize edilmiş sekant hiperbolik fonksiyonu.

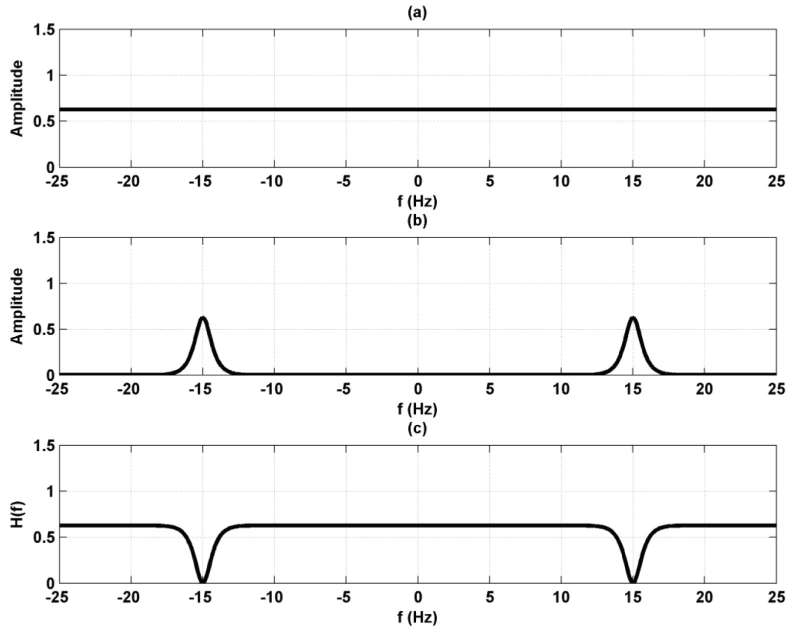

Figure 3. (a) A constant value, (b) The behavior of the filter characteristic for $c=5$, (c) The difference between constant and two secant hyperbolic functions which give an expression of a notch filter in the frequency domain.

Şekil 3. (a) Sabit değer, (b) c=5 için süzgeç karakteristiği değişimini, (c) Sabit bir değer ve iki sekant fonksiyonu arasındaki fark frekans ortamında çentik süzgeci fonksiyonunu vermektedir.

$$h(t) = A\delta(t) - 2\cos(2\pi f_c t)\operatorname{sec}h(c\,t) \quad (6)$$

Equation 6 is called the impulse response of the filter. To calculate notch filter coefficients, Equation 6 is convolved with a sin c function. Thus, the result can be written

$$b_n(t) = h(t) * \sin c \quad (7)$$

where $*$ stands for the convolution process. Substitute Equation 6 into Equation 7 which yields

$$b_n(t) = \left(A\delta(t) - 2\cos(2\pi f_c t)\operatorname{sec}h(c\,t)\right) * \sin c \quad (8)$$

where the sinc function is given by $\sin c = \sin(2\pi f_N t) / 2\pi f_N t$. Finally, the filter coefficients of a notch filter is calculated by using the following expression

$$b_n(n\Delta t) = A\frac{\sin(2\pi f_N n\Delta t)}{2\pi f_N n\Delta t} - \frac{1}{f_N}\cos(2\pi f_c n\Delta t)\operatorname{sec}h(c\,n\Delta t) \quad (9)$$

where f_N is the Nyquist frequency and f_c is the cut-off frequency. Δt is the time-step size with a corresponding index number n. From Equation 9, one can calculate notch filter coefficients. Equation 9 has a singularity point at t=0. So it is evaluated by using the limit theorem (Abromowitz and Stegun, 1965). When $\lim_{x\to 0} \sin(x)/x = 1$ considered, the following expression is used for estimating b(0) filter coefficient:

$$b_n(0) = A - \frac{1}{f_N} \quad (10)$$

The rest of the coefficients is calculated by using Equation 9, except at t=0 point.

Cascaded-Notch Filter Design

The method used in the previous section may be extended to cascaded-notch filter design. It is possible to filter out a couple of harmonics from a data set rather than one specific frequency. Cascading helps in designing a filter for that kind of purpose. The idea of what have been done so far in the previous section can be extended then in the design of a cascaded-filter.

Suppose that, data are contaminated with f_1 and f_2 frequencies. These frequencies need to be filtered out from the data. In this case, the frequency response can be derived as

$$H(f) = A - \left\{ \left(\frac{\pi}{c}\right)\mathrm{sec}\,h\left(\frac{\pi^2(f+f_1)}{c}\right) + \left(\frac{\pi}{c}\right)\mathrm{sec}\,h\left(\frac{\pi^2(f-f_1)}{c}\right) + \left(\frac{\pi}{c}\right)\mathrm{sec}\,h\left(\frac{\pi^2(f+f_2)}{c}\right) + \left(\frac{\pi}{c}\right)\mathrm{sec}\,h\left(\frac{\pi^2(f-f_2)}{c}\right) \right\} \quad (11)$$

where $A = \frac{\pi}{c}$. An expression of the filter coefficient is obtained by using the same process as in the previous section. After a straightforward derivation, the filter coefficients are calculated by

$$b_{cas}(n\Delta t) = A\frac{\sin(2\pi f_N n\Delta t)}{2\pi f_N n\Delta t} - \frac{1}{f_N}\left(\cos(2\pi f_1 n\Delta t) + \cos(2\pi f_2 n\Delta t)\right)\mathrm{sec}\,h(c\, n\Delta t) \quad (12)$$

Equation 12 can be written in a more general case. Thus, a general expression for cascaded-notch filter can be

$$b_G(n\Delta t) = A\frac{\sin(2\pi f_N n\Delta t)}{2\pi f_N n\Delta t} - \frac{1}{f_N}\left(\sum_{m=1}^{M}\cos(2\pi f_m n\Delta t)\right)\mathrm{sec}\,h(c\, n\Delta t) \quad (13)$$

where m is an index number and M is a number of frequencies which needs to be eliminated from the corresponding signal.

Comparison of Some Common Windows

In signal processing, windowing is used for filter design and reducing the spectral leakage from a signal. The main purpose of using windows for the filter design is to increase the main-lobe and reducing the side-lobe effects of a signal (Hayes, 1999). Here, the comparison was conducted the results obtained with some common windows in signal processing such as Bartlett, Parzen, Tukey, and Kaiser. Figure 4 shows the filter coefficient calculated using these windows. One can see that the filter coefficients illustrated in Figure 4 are very close to each other's. To distinguish the differences between these results, the relative errors are shown in Figure 5. From these numerical calculations, it can be figured out that the signal filtered using Tukey and Kaiser windows gave better results based on the relative errors. The formulas used in this study for Bartlett, Parzen, Tukey, and Kaiser windows are given in Appendix.

In general, the signal is multiplied by some windows before Fourier transform to reduce the spectral leakage. The spectral leakage occurs, since the signal is measured some internal without knowing the exact period of events (Ertürk, 2009). Figure 6 illustrates effects of windows on the signal. From these numerical experiments, it is easy to see that the Tukey and Kaiser windows reduce the spectral leakage effect significantly. However, Bartlett and Parzen windows cannot reduce the spectral leakage successfully.

NUMERICAL EXAMPLES

Time domain

The applicability of the notch filter was tested by using a simple numerical example. Figure 7 (a) illustrates the signal. The signal consists of two harmonics at 0.5 and 5 Hz frequencies. The synthetic data in Figure 7 (a) is generated by

$$g(t) = 1.5\cos(t\pi) + 2.5\cos(10\,t\pi) \quad (14)$$

where t is a time-step size. The time-step size is set to 0.01 s. The range of the harmonics is -5 to 5 s. The Nyquist frequency for this signal is set to 50 Hz and *c* constant value is chosen to be 6.7. Trial and error methods are a way to get an optimum number of filter coefficients. Here, it has been used trial and error methods to get an optimum c value. The number of filter coefficients (or filter weights) is chosen 101.

After calculating filter coefficients, the signal is convolved with them. Figure 7 (b) shows the filter coefficients. The original synthetic data has two harmonics. The signal at 5 Hz needs to be eliminated from the original synthetic data. To achieve this, the original synthetic data is convolved with the filter coefficients. Figure 7 (c) displays the result. The relative error of the notch filter is given in Figure 8.
The second numerical example is a notch filter by cascading. The synthetic data can be generated by

$$g_{cas}(t) = 2.5\cos(5\,t\pi) + 1.5\cos(10\,t\pi) + \cos(30\,t\pi) \quad (15)$$

where t is the time (s) on the interval between -5 to 5 s with $\Delta t = 0.01$. The original signals consist of three harmonics. The frequencies are at 2.5, 5 and 15 Hz. The harmonics at 5 and 15 Hz (unwanted-signals, noise) needs to be eliminated from the corresponding data set. The original signals are shown in Figure 9 (a). The filter coefficients are displayed in Figure 9 (b). The bottom panel of Figure 9 (c) shows the filtered data by using the cascaded-notch filter. A notch filter designed by using the hyperbolic secant function was successfully applied to the signal and

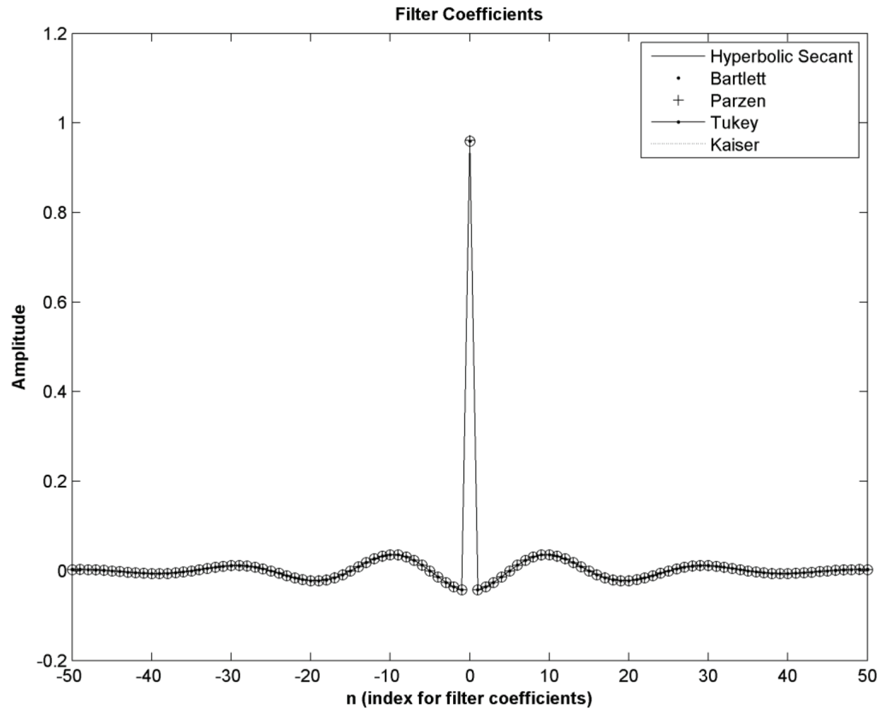

Figure 4. Numerical comparison of Bartlett, Parzen, Tukey and Kasier windows and Hyperbolic secant function.
Şekil 4. Bartlett, Parzen, Tukey ve Kasier pencerelerinin Hiperbolik sekant fonksiyonu ile sayısal karşılaştırılması.

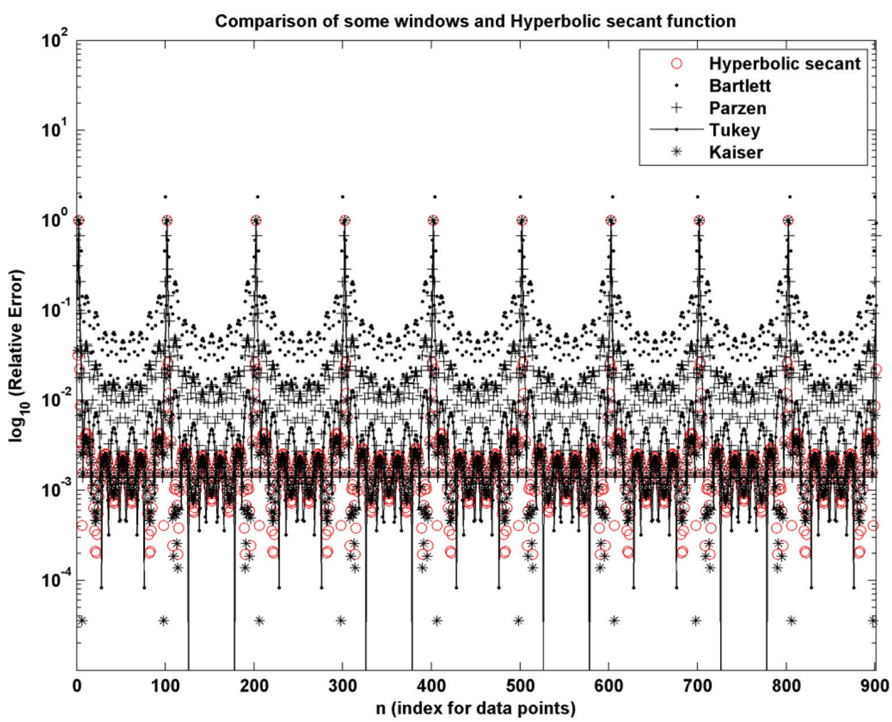

Figure 5. Relative errors of Hyperbolic secant function, Bartlett, Parzen, Tukey, and Kaiser windows.
Şekil 5. Hiperbolik sekant fonksiyonu, Bartlett, Parzen, Tukey, ve Kaiser pencerelerinin göreceli hataları.

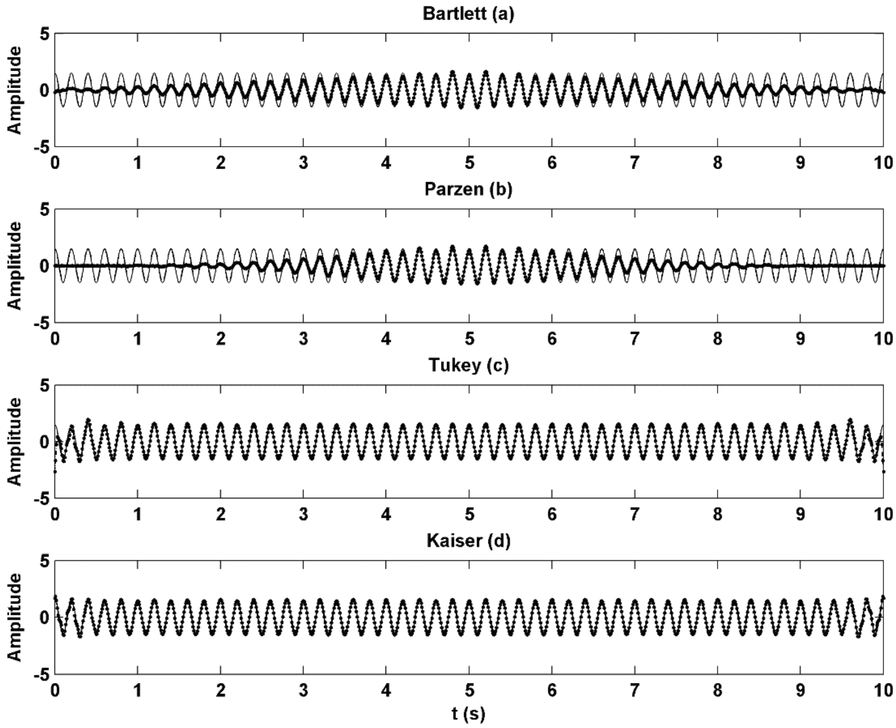

Figure 6. Comparison of Bartlett (a), Parzen (b), Tukey (c) and Kaiser (d) windows. The solid lines show the original signal. The dotted lines display the filtered data for various windows.

Şekil 6. Bartlett (a) , Parzen (b), Tukey (c) ve Kaiser (d) pencerelerinin karşılaştırılması. Sürekli çizgi orijinal veriyi göstermektedir. Noktalı çizgiler farklı pencerelerle süzülmüş verileri göstermektedir.

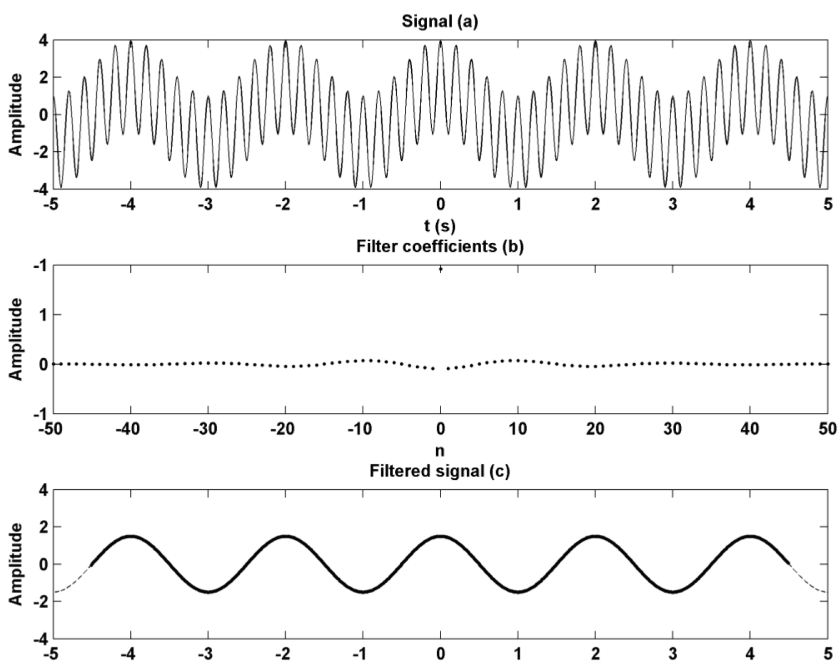

Figure 7. (a) The signal generated by harmonic functions frequencies at 0.5 and 5 Hz. (b) Filter weights, and (c) The signal filtered out by the notch filter. Panel (c) shows the original signal (dashed lines) and filtered signal (dotted lines).

Şekil 7. (a) 0.5 ve 5 Hz frekanslı harmonik fonksiyonlardan üretilen sinyal. (b) Süzgeç katsayıları ve (c) Çentik süzgeç ile süzülmüş sinyal.Orijinal sinyal (kesikli çizgiler) ve süzülmüş sinyal (noktalı çizgiler).

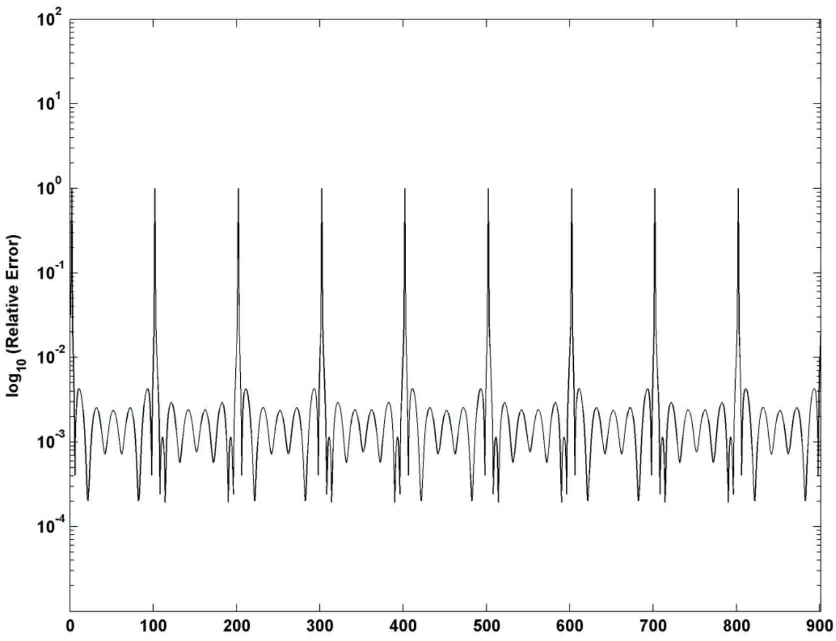

Figure 8. The relative errors of the notch filter.
Şekil 8. Çentik süzgecin göreceli hatası.

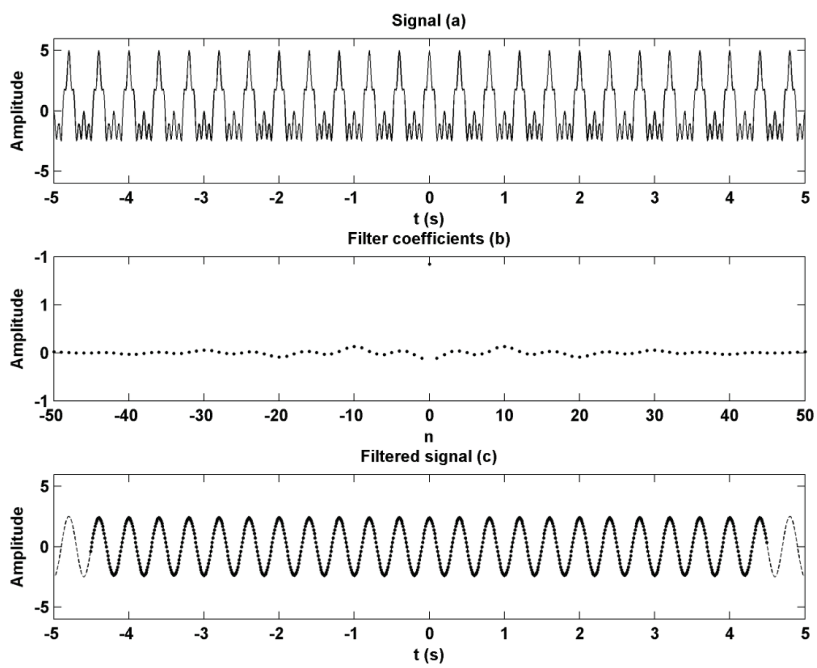

Figure 9. A notch filter applied of a signal is shown. (a) The signal generated by harmonic functions at 2.5, 5, and 15 Hz. (b) Filter weights is shown in the middle panel, and (c) The signal can be filtered out by the notch filter. Panel (c) shows the original signal (dashed lines) and filtered signal (dotted lines).
Şekil 9. Çentik süzgeç uygulanmış bir sinyal gösterilmektedir (a) 2.5, 5 ve 15 Hz frekanslı harmonik fonksiyonlardan üretilen sinyal (b) Filtre katsayıları orta panelde gösterilmektedir. Sinyal çentik süzgeç ile süzülebilir. (c) Orijinal sinyal (kesikli çizgiler) ve süzülmüş sinyal (noktalı çizgiler).

the signal was filtered out two frequencies from the data.

Comparison of band-stop and notch filters

The band-stop filter can also be used for filtering from the signal at a specific frequency. However, the proposed notch filter has some advantages over the band-stop filter. To illustrate these advantages, a synthetic signal generated at 2.5, 5 and 15 Hz can be used. If a band-stop filter is used to eliminate 5 Hz harmonics from the generated signal, it is necessary to use more filter coefficients to achieve the same accuracy with the result obtained the proposed filter. Figure 10 shows the filtered result using the band-stop and the hyperbolic notch filter together. The parameters used for the synthetic signal are the same with the previous example. The cut-off frequency is 5 Hz for the hyperbolic notch filter. As for the band-stop filter, the cut-off frequencies are 4.9 and 5.1, respectively. The reader is refered to Başokur (2007) for more detail information about the band-stop filter.

Frequency domain

So far we have done the processes in the time domain which can also be applied to the frequency domain as well. Figure 11 (a) shows a synthetic data set generated by Equation 15. Equation 15 in the frequency domain is given

$$G(f)=\frac{2.5}{2}\delta(f+2.5)+\frac{2.5}{2}\delta(f-2.5)+\frac{1.5}{2}\delta(f-5)+\frac{1.5}{2}\delta(f+5)+\frac{1}{2}\delta(f-15)+\frac{1}{2}\delta(f+15) \quad (16)$$

Equation 16 is shown in Figure 11 (b). To eliminate noise from the data, the frequency response is multiplied by the FFT of the signal (Figure 11c). The result of the multiplication gives the filtered data in the frequency domain (Figure 11d). To obtain the data in the time domain, we carry out the Inverse Fast Fourier Transform (IFFT) to the data (Figure 11e). Figure 11 a-e shows the result of a numerical example of the cascaded-notch filter in the frequency domain step by step. In this numerical example we used the same parameters with the previous signal.

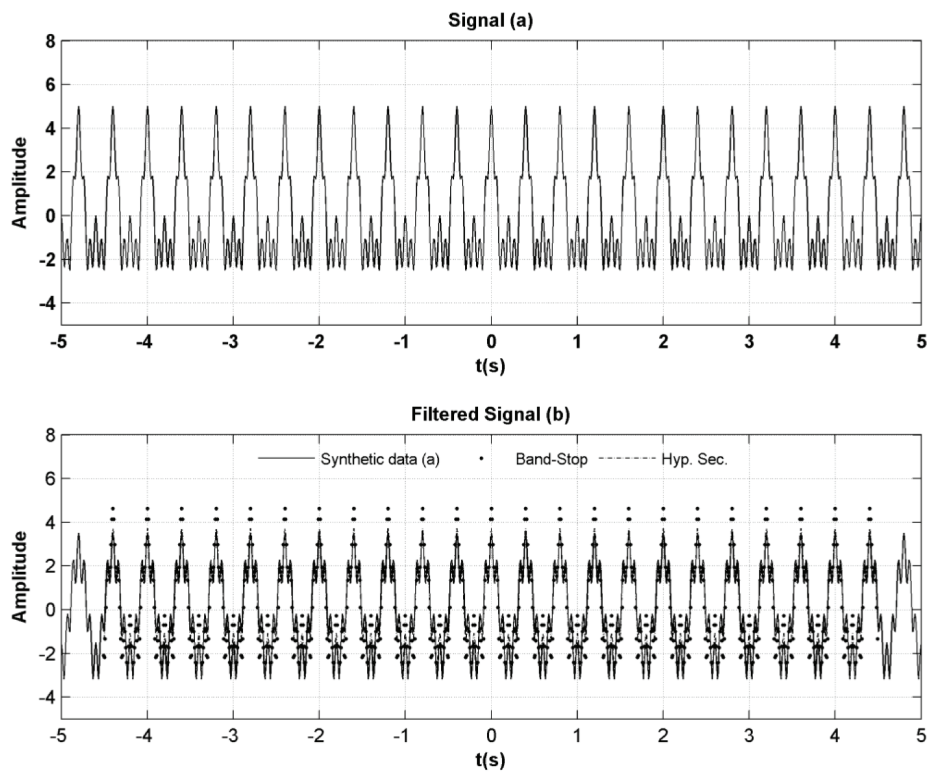

Figure 10. Panel (a) shows the signal. Panel (b) shows the comparison of band-stop and hyperbolic secant filter.
Şekil 10. (a) paneli sinyali göstermektedir. (b) paneli bant-durdurucu ve hiperbolik sekant süzgeç karşılaştırılması gösterilmektedir.

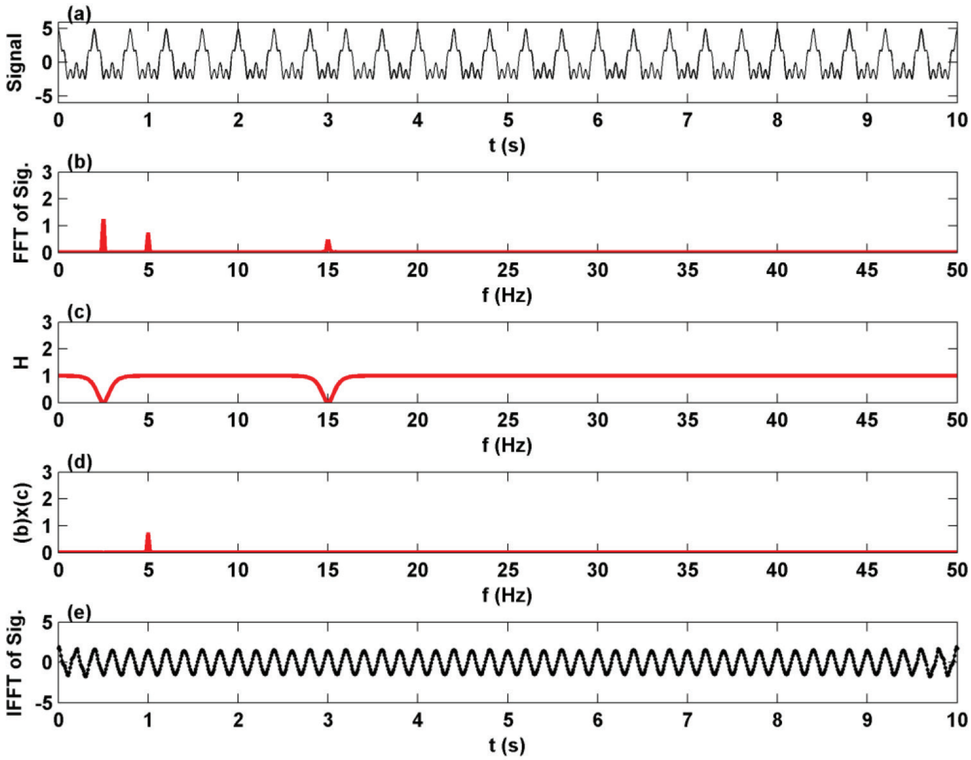

Figure 11. The frequency domain application of a notch filter. Panel (a) shows the signal. The FFT of the signal is illustrated in (b). The filter characteristic is given in Panel (c). (d) is obtained by multiplication of (b) and (c). The IFFT of the (d) is shown in Panel (e).

Şekil 11. Çentik süzgecinin frekans ortamı uygulaması. (a) paneli sinyali göstermektedir. Sinyalin Fourier dönüşümü (b) sunulmuştur. Filtre karakteristiği (c) de verilmektedir. (d) şekli (b) ve (c) nin çarpımından elde edilir. (e) ise (d) nin ters Fourier dönüşümünü göstermektedir.

FIELD EXAMPLE, DIGITAL FILTERING ON SEISMOGRAMS

In order to test the proposed notch filter performance, a set of field measurement were used. The data recorded with REFTEK seismometer with 3 components broad band device. Figure 12 shows Z (vertical) components with 1800 s measured using a seismometer. Time step is 0.01 s. Figure 12 (a) displays the seismogram in the time domain. The data were multiplied a Kaiser window to reduce the spectral leakage before filtering. The same data set are displayed in frequency domain (Figure 12b). It could be seen that, there could be interference at around 50 Hz due to the power-line sources. There is a rectangular shape with dashed lines in the middle of Figure 12 (b), which shows noise. After multiplying the signal in frequency domain with the frequency response of the proposed filter, frequencies around 50 Hz were successfully filtered out (See Figure 12c). The bottom panel in Figure 12 (d) illustrates inverse Fourier transform of the filtered signal. Figure 13 shows a closer look at the interference around 50 Hz.

CONCLUSION

An analytic expression for notch filter was derived using the hyperbolic secant function and the possibility of cascaded-notch filters was shown. The design of digital – notch filter using the secant hyperbolic function was demonstrated and was applied to the signal for filtering out some specific frequencies from the data set. These filters might be used in geophysics, biomedical sciences and other related sciences for filtering one or more specific frequencies (from sources such as power–lines and other sources) from contaminated data set. The notch filter designed in this study is analytical one and since the

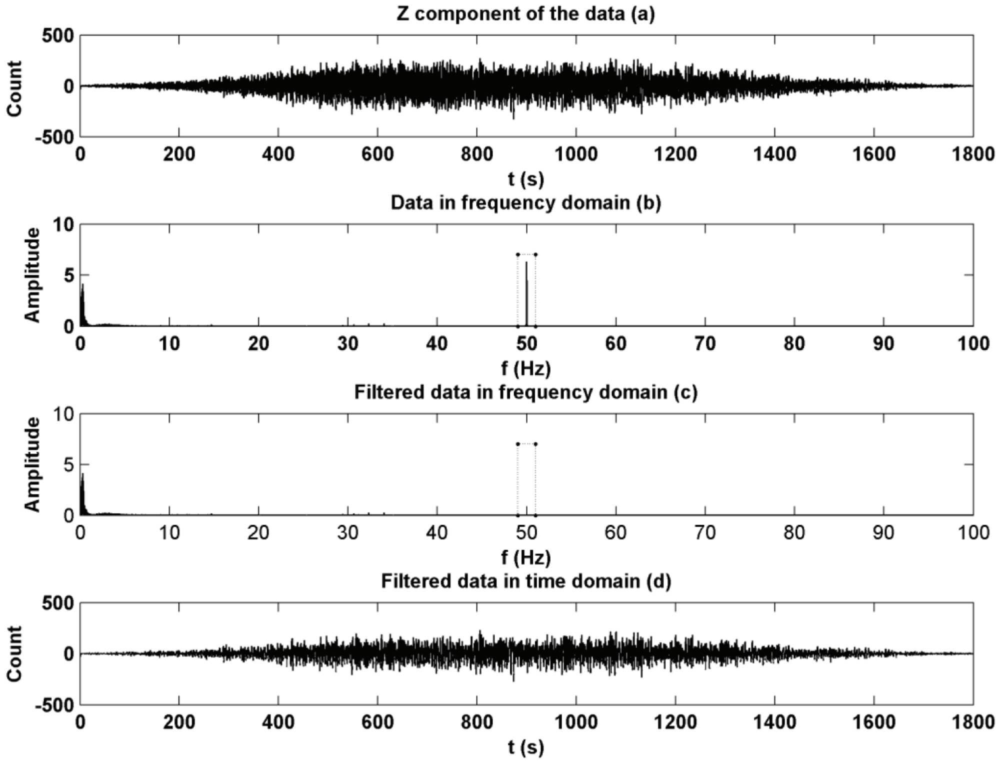

Figure 12. A seismogram (Z component) (a). (b) FFT of field data. (c) The filtered signal. (d) IFFT of the filtered signal. Rectangular areas with dashed lines in (b) and (c) emphasize before and after filtering the data.
Şekil 12. Sismogram (Z bileşeni) (a). (b) Arazi verisinin Fourier dönüşümü. (c) Süzülmüş sinyal. (d) Süzülmüş sinyalin ters Fourier dönüşümü. (b) ve (c) de kesikli dikdörtgen alanlar verinin filtrelenmeden önceki ve sonraki önemini göstermektedir.

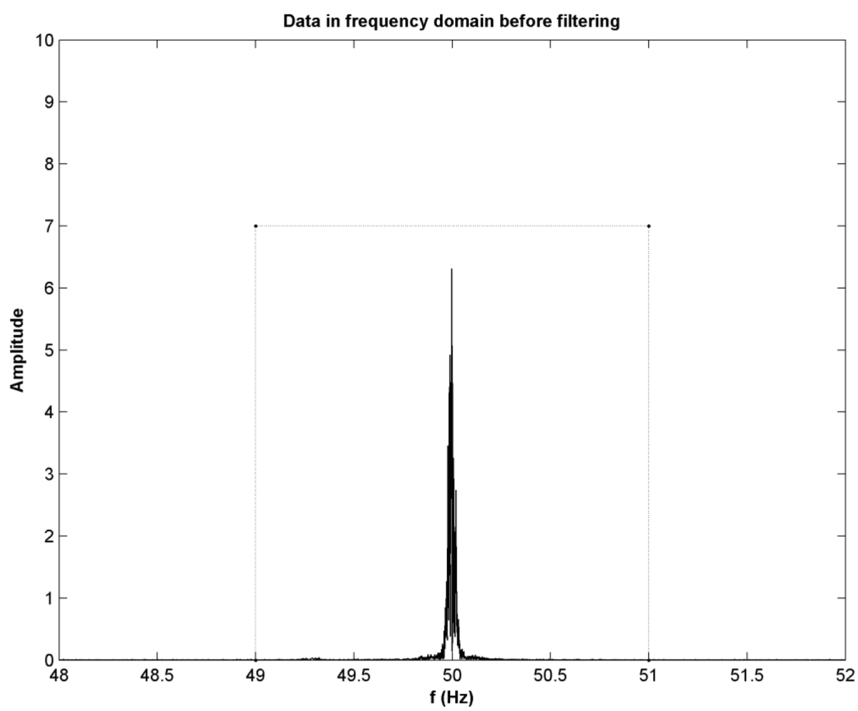

Figure 13. Noisy data before filtering around 50 Hz.
Şekil 13. Filtreleme öncesi 50 Hz civarında gürültülü data.

hyperbolic secant function has no singularity point within plus and mines infinity, the filter is stable.

The proposed filter has been compared with some common windows used in signal processing. The proposed signal derived a smooth hyperbolic-secant filter compared Bartlett, Parzen, Tukey, and Kaiser windows. The proposed filter gives a better result than Bartlett and Parzen do. However, there have been very small differences observed among the proposed filter Tukey and Kaiser windows. We also compared the band-stop filter with the proposed filter. The result obtained with the proposed filter much better than the result obtained with the band-stop filter. The proposed filter can be used in seismic data set as well. The seismic data set for oil exploration often is contaminated with power-lines.

As a conclusion, the new filter coefficient obtained by using the hyperbolic secant function has successfully been applied to a synthetic data set. And the filter has also been tested on the field data set as well.

ACKNOWLEDGEMENT

The authors would like to thank Mehmet Küçüktiftik from Erkoç Elektrik Company for providing the instantaneous value of power-line frequency variation. The authors also would like to thank to TUBITAK MAM for providing the data set for the study.

REFERENCES

Abromowitz, M., and Stegun, I.A., 1965. Handbook of mathematical functions. Dover Publications Inc..

Başokur, A.T., 1998. Digital filter design using the hyperbolic tangent functions. Journal of the Balkan Geophysical Society, 1, 14-15.

Başokur, A.T., 2007. Spectral Analiz and Sayısal Süzgeçler. JFMO Eğitim Yayınları No:8 (in Turkish).

Başokur, A.T., 2011. Designing Frequency Selective Filters Via the Use of Hyperbolic Tangent Functions. Bulletin of Earth Sciences Application and Research Centre of Hacettepe University, 1, 69-88.

Bloomfield , P., 2000. Fourier Analysis of Time Series: An Introduction. New York: Wiley-Interscience, p.69.

Bracewell, R., 1965. The Fourier transform and its applications. McGraw_Hill, Inc..

Butler, K.E., and Russell, R.D., 1993. Subtraction of powerline harmonics from geophysical records. Geophysics, 58, 898-903.

Cadzow, J.A., 1973. Discrete-Time systems: An introduction with interdisciplinary applications. Prentice—Hall, Inc.

Deshpande, R., Jain, S.B., and Kumar, B., 2008. Design of maximally flat linear phase FIR notch filter with controlled null width. Signal Processing, 88, 2584-2592.

Ertürk, S., 2009. Sayısal işaret işleme, Birsen Yayınevi (in Turkish).

Gradshteyn, I.S., and Ryzhik, I.M., 1994. Table of integrals, series, and products, Academic Press Inc.

Harris, F.J., 1978. On the Use of Windows for Harmonic Analysis with the Discrete Fourier Transform. Proceedings of the IEEE, 6651-83.

Hattingh, M., 1988. A new data adaptive filtering program to remove noise form geophysical time- or space- series data. Computers & Geosciences, 14, 467-480.

Hayes, M.N., 1999. Digital Signal Processing, Schaum's Outline Series, McGraw-Hill. p.361.

Haykin, S, 1991. Adaptive filter design, Prentice Hall.

Johensen, H.K., and Sorensen, K., 1979. Fast Hankel transforms. Geophysical Prospecting, 27, 876-901.

Oppenheim, A.V., and R.W. Schafer. 1999. Discrete-Time Signal Processing. Upper Saddle River, NJ: Prentice-Hall, 468-471.

Piskorowski, J., 2012. Suppressing harmonic powerline interference using multiple-notch filtering methods with improved transient behavior. Measurement, 45, 1350-1361.

Piskorowski, J., 2013. Time-efficient removel of power-line noise from EMG signals using IIR notch filters with non-zero initial conditions. Bioceybernetics and Biomedical Engineering, 33, 1771-178.

Stancic, G., and Nikolic, S., 2013. Digital linear phase notch filter design based on IIR all-pass filter applications. Digital Signal Processing, 23, 1065-1069.

APPENDIX

Some Common Windows

In signal processing, windowing is used for increasing the main-lobe and reducing the side-lobe effects of the signal as mentioned in previous. Here, formulas used for Bartlett, Parzen, Tukey, and Kaiser windows are introduced. A triangular shape of a Bartlett can be calculated using as the following expression

$$w_1(n) = \begin{cases} \frac{2n}{\mathrm{N}}, \ 0 \le \mathrm{n} \le \frac{\mathrm{N}}{2} \\ 2 - \frac{2n}{\mathrm{N}}, \ \frac{\mathrm{N}}{2} \le \mathrm{n} \le \mathrm{N} \end{cases}$$

where n is an index. N is the number of points calculated (Oppenheim and Schafer, 1999). A similar expression for Parzen window is given

$$w_2(n) = \begin{cases} 1 - 6\left(\frac{|n|}{N/2}\right)^2 + 6\left(\frac{|n|}{N/2}\right)^3, 0 \le |\mathrm{n}| \le (N-1)/4 \\ 2\left(1 - \frac{|n|}{N/2}\right)^3, (N-1)/4 < |\mathrm{n}| \le (N-1)/2 \end{cases}$$

(Harris, 1978). Tukey windows is

$$w_3(x) = \begin{cases} \frac{1}{2}\left\{1 + \cos\left(\frac{2\pi}{\alpha}\left[x - \frac{\alpha}{2}\right]\right)\right\}, \ 0 \le \mathrm{x} \le \frac{\alpha}{2} \\ 1, 0 \le \mathrm{x} \le \frac{\alpha}{2} \\ \frac{1}{2}\left\{1 + \cos\left(\frac{2\pi}{\alpha}\left[x - 1 + \frac{\alpha}{2}\right]\right)\right\}, \ 1 - \frac{\alpha}{2} \le \mathrm{x} \le 1 \end{cases}$$

where x is a N-point linearly spaced vector. The parameter α is the ratio of cosine-tapered section length to the entire window length with. Here, $\alpha = 0.5$ is used (Bloomfield, 2000). Kaiser developed a family of windows can be generated with

$$w_4(n) = \frac{I_0\left[\beta\left(1 - \left[{(n-\alpha)}/{\alpha}\right]^2\right)^{1/2}\right]}{I_0}$$

where $\alpha = N/2$ and I_0 is a zero order Bessel function of the first kind. The parameter β controls the shape of the window. Therefore, it may be used for the trade-off between main-lobe width and side-lobe amplitude. To apply a window to the filter, as the following expression can be used

$$b(n) = b_n(n) w_m(n)$$

where m stands for window types with 1, 2, 3, and 4. Namely, proposed filter is multiplied with a corresponding window type.